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A NEW GENERALIZATION OF δ-CLOSED SETS USING TWO DIFFERENT OPERATORS

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ABSTRACT

The scope of this paper is to give a new generalization to δ-closed sets namely λ_g^δ-closed sets which involves the use of two different operators. Some interesting theorems involving λ_g^δ-closed sets are also discussed.

KEYWORDS: δ-closed sets, δg-closed sets, gδ-closed sets,δg*-closed sets,(Λ, δ)-closed sets,λ_g^δ-closed sets

INTRODUCTION

Ever since the notion of δ-closed sets in topological spaces was introduced by Velicko in 1968, several authors started to extend this concept via various types of generalizations[1][2][3][4][5][9][12]. As an outcome of these generalizations, various forms of closed sets and interesting separation axioms came into existence. In 2004, Georgiou discussed an unique type of generalization of δ-closed set namely (Λ, δ)-closed set which is defined as the intersection of a Λ_δ -set and a δ-closed set. A subset A of a topological space (X, τ) is called a Λ_δ -set if Λ_δ(A)=A, where Λ_δ(A) is the intersection of allδ-open sets containing A. Λ_δ plays the role of an operator which is an alternative to the classical closure operator. In this paper, we have portrayed a new variety of generalization involving the classical closure operator and the operator Λ which is defined to be the intersection of all open sets containing A. The outcome of such a generalization is named to be λ_g^δ-closed sets. We have discussed the relationship of λ_g^δ-closed sets with some already existing sets in the literature followed by some interesting characterizations using few already existing spaces. Finally, the notion of λ_g^δ-open sets and some properties are discussed.

PREREQUISITES

Definition 2.1: A subset A of a topological space (X, τ) is called

(1) **regular open**[11] if int(cl(A))=A.
(2) **δ-open**[14] if it is the union of regular open sets, the complement of δ-open is called δ-closed.
(3) **Λ_δ -set**[5] if Λ_δ(A)=A, where $\Lambda_\delta(A)=\cap\{O\in\delta O(X,\tau) \mid A\subseteq O\}$.
(4) **(Λ, δ)-closed**[5] if A = T∩C, where T is a Λ_δ -set and C is a δ-closed set.
(5) **δg-closed**[2] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ)
(6) **δg*-closed**[3]if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is δ-open in (X, τ).
(7) **gδ-closed**[3]if cl(A) ⊆ U whenever A ⊆ U and U is δ-open in (X, τ).
(8) **δgs-closed**[6]if δ-scl(A) ⊆ U whenever A ⊆ U and U is δ-open in (X, τ).
(9) **gδs-closed**[1]if scl(A) ⊆ U whenever A ⊆ U and U is δ-open in (X, τ).

Remark: The class of all regular closed(resp. δ-closed, (Λ, δ)-closed) sets are denoted by RC(X,τ)(resp. δC(X, τ), (Λ, δ)C(X, τ)).

Definition 2.2: A subset A of a topological space (X, τ) is called
(1) **weakly Hausdorff**[3] if every singleton is δ-closed.

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(2) **$T_{3/4}$ -space**[2]if every δg-closed set is δ-closed in (X, τ).
(3) **δ-door space**[13] if everysubset of (X, τ) is either δ-open or δ-closed in (X, τ).
(4) **T_δ -space**[3]if every gδ-closed set is δ-closed in (X, τ).
(5) **$_\delta T_{3/4}$ -space**[1]if every gδs-closed set is δ-closed in (X, τ).

λ_g^δ-CLOSED SETS IN TOPOLOGICAL SPACES

Definition 3.1: A subset A of a topological space (X, τ) is called **λ_g^δ-closed set** if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is ($\wedge$, δ)-open in X. The family of all λ_g^δ-closed sets of (X, τ) is denoted by $\lambda_g^\delta C(X, \tau)$.

Proposition 3.2: Every δ-closed set is λ_g^δ-closed but not conversely.

Proof: Let A be a δ-closed set and U be a ($\wedge$, δ)-open set containing A. Since A is δ-closed, $cl_\delta(A) = A$. Therefore $cl_\delta(A) = A \subseteq U$ and hence A is λ_g^δ-closed.

Counter example 3.3: Let X = {a, b, c}, τ = {X, ϕ, {a}}. Take A = {a} then A is λ_g^δ-closed but not δ-closed as $\delta C(X, \tau)$ = {X, ϕ}.

Proposition 3.4: Every λ_g^δ-closed set is δg^*-closed but not conversely.

Proof: Let A be a λ_g^δ-closed set and U be a δ-open set containing A. Since every δ-open set is ($\wedge$, δ)-open[5] and A is λ_g^δ-closed, $cl_\delta(A) \subseteq U$. Therefore A is δg^*-closed.

Counter example 3.5: Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {c}, {a, b}, {a, c}, {a, b, c}, {a, c, d}}. Take A = {a, c} then A is δg^*-closed but not λ_g^δ-closed.

Proposition 3.6: Every λ_g^δ-closed set is gδ-closed but not conversely.

Proof: Since $cl(A) \subseteq cl_\delta(A)$, the proof follows that of Proposition 3.4.

Counter example 3.7: Let X and τ be defined as in Example 3.5. Take A = {b} then A is gδ-closed but not λ_g^δ-closed.

Proposition 3.8: Every λ_g^δ-closed set is δgs-closed but not conversely.

Proof: Since $\delta\text{-}scl(A) \subseteq cl_\delta(A)$, the proof follows that of Proposition 3.4.

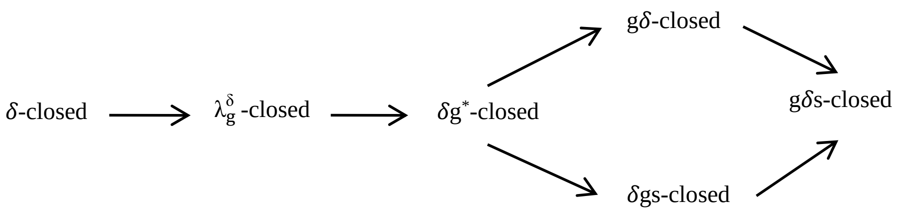
Counter example 3.9: Let X, τ and A be defined as in Example 3.7 then A is δgs-closed but not λ_g^δ-closed.

Proposition 3.10: Every λ_g^δ-closed set is gδs-closed but not conversely.

Proof: Since $scl(A) \subseteq cl_\delta(A)$, the proof follows that of Proposition 3.4.

Counter example 3.11: Let X, τ and A be defined as in Example 3.7 then A is gδs-closed but not λ_g^δ-closed.

The newly defined family of λ_g^δ-closed sets properly fits between the family of δ-closed sets and δg^*-closed sets as observed from the following link.



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Remark 3.12: Closedness(resp. g-closedness, α-closedness, semi-closedness, pre-closedness, δg-closednes) is independent of λ_g^δ-closedness as observed from the following examples.

Example 3.13: Let X = {a, b, c} and τ = {X, ϕ, {a}, {c}, {a, b}, {a, c}, {a, b, c}, {a, c, d}}. Take A = {a, b, d} then A is closed(resp. g-closed, α-closed, semi-closed, pre-closed and δg-closed) but not λ_g^δ-closed in (X, τ).

Example 3.14: Let X = {a, b, c} and τ = {X, ϕ, {a}}. Take A = {a} then A is λ_g^δ-closed but not closed(resp. g-closed, α-closed, semi-closed, pre-closed and δg-closed) in (X, τ).

Theorem 3.15: Let A be a λ_g^δ-closed set in (X, τ). Then cl_δ(A)-A does not contain a non-empty (Λ, δ)-closed set.

Proof: Suppose that A is λ_g^δ-closed and let F be a (Λ, δ)-closed set contained in cl_δ(A)-A. Now F^c is a (Λ, δ)-open set in X such that A ⊆ F^c. Since A is a λ_g^δ-closed, cl_δ(A) ⊆ F^c. Thus F ⊆ $(cl_\delta(A))^c$. Also F ⊆ cl_δ(A)-A. Therefore F ⊆ $(cl_\delta(A))^c \cap cl_\delta(A)$ = ϕ. Hence F = ϕ.

Proposition 3.16: If A is a (Λ, δ)-open set and a λ_g^δ-closed set of (X, τ) then A is a δ-closed set of X.

Proof: Since A is (Λ, δ)-open and λ_g^δ-closed, cl_δ(A) ⊆ A. Hence A is δ-closed.

Theorem 3.17: If A is a λ_g^δ-closed set and (Λ, δ)-open and F is δ-closed in (X, τ), then A⋂F is δ-closed.

Proof: Since A is λ_g^δ-closed and (Λ, δ)-open, A is δ-closed by Proposition 3.16. Since F isδ-closed in X, A⋂F is δ-closed in X.

Proposition 3.18: If A is a λ_g^δ-closed set in (X, τ) and A ⊆ B ⊆ cl_δ(A), then B is also a λ_g^δ-closed set.

Proof: Let U be a (Λ, δ)-open set of X containing B. Then A ⊆ U. Since A is λ_g^δ-closed, cl_δ(A) ⊆ U. Also since B ⊆ cl_δ(A), cl_δ(B) ⊆ $cl_\delta(cl_\delta(A))$ = cl_δ(A). Hence cl_δ(B) ⊆ U and therefore B is λ_g^δ-closed.

Theorem 3.19: Let A be a λ_g^δ-closed set of (X, τ). Then A is δ-closed iff cl_δ(A)-A is (Λ, δ)-closed.

Proof: Necessity: Let A be a δ-closed subset of (X, τ). Then cl_δ(A) = A and so cl_δ(A)-A = ϕ, which is (Λ, δ)-closed.
Sufficiency: Let cl_δ(A)-A be (Λ, δ)-closed. Since A is λ_g^δ-closed, by Theorem 3.15, cl_δ(A)-A does not contain a non-empty (Λ, δ)-closed set which implies cl_δ(A)-A = ϕ. Therefore cl_δ(A) = A and hence A is δ-closed.

Characterizations of λ_g^δ-closed sets through existing spaces

Proposition 3.20: In a $T_{3/4}$ -space, every δg-closed set is λ_g^δ-closed.

Proof: Let (X, τ) be a $T_{3/4}$ -space and let A be a δg-closed set in (X, τ). In $T_{3/4}$ -space, the class of all δg-closed sets coincide with that of δ-closed sets. Therefore A is δ-closed in (X, τ). Moreover, every δ-closed set is λ_g^δ-closed and hence A is λ_g^δ-closed in (X, τ).

Proposition 3.21: In a δ-door space, every subset is either λ_g^δ-open or λ_g^δ-closed.

Proof: Let A be a subset of (X, τ) which is a δ-door space. Then every subset is either δ-open or δ-closed. Since every δ-open or δ-closed is λ_g^δ-open or λ_g^δ-closed respectively, we have A is either λ_g^δ-open or λ_g^δ-closed.

Proposition 3.22: In a weakly Hausdorff space, every singleton is λ_g^δ-closed.

Proof: Let (X, τ) be a weakly Hausdorff space. Then every singleton is δ-closed. Since every δ-closed set is λ_g^δ-closed, we have that every singleton is λ_g^δ-closed.

Proposition 3.23: The family of all gδ-closed sets and that of λ_g^δ-closed sets coincide in a T_δ -space.

Proof: Let A be a λ_g^δ-closed set. By Proposition 3.6, every λ_g^δ-closed set is a gδ-closed set. Therefore A is gδ-closed. Conversely, let A be gδ-closed. In T_δ -space, every gδ-closed set is δ-closed. Since every δ-closed set is λ_g^δ-closed, we have A is λ_g^δ-closed.

Proposition 3.24: In a ${}_\delta T_{3/4}$ -space, the family of all gδs-closed sets and that of λ_g^δ-closed sets coincide.

Proof: Let A be a λ_g^δ-closed set. By Proposition 3.10, every λ_g^δ-closed set is a gδs-closed set. Therefore A is gδs-closed. Conversely, let A be gδs-closed. In ${}_\delta T_{3/4}$ -space, every gδs-closed set is δ-closed. Since every δ-closed set is λ_g^δ-closed, we have A is λ_g^δ-closed.

4. λ_g^δ-OPEN SETS IN TOPOLOGICAL SPACES

Definition 4.1: A subset A of a topological space (X, τ) is called **λ_g^δ-open** if its complement A^c is λ_g^δ-closed in (X, τ). The family of all λ_g^δ-open sets in (X, τ) is denoted by $\lambda_g^\delta O(X, \tau)$.

Lemma 4.2:[12] For a subset A of (X, τ), $cl_\delta(X \backslash A) = X \backslash int_\delta(A)$.

Theorem 4.3: A subset A of a topological space (X, τ) is λ_g^δ-open if and only if G ⊆ $int_\delta(A)$ whenever G ⊆ A and G is (Λ, δ)-closed.

Proof: Necessity: Assume that A is λ_g^δ-open. Then A^c is λ_g^δ-closed. Let G be a (Λ, δ)-closed set in (X, τ) such that G ⊆ A. Then G^c is (Λ, δ)-open in (X, τ) such that $A^c \subseteq G^c$. Since A^c isλ_g^δ-closed, $cl_\delta(A^c) \subseteq G^c$, equivalently $G \subseteq int_\delta(A^c)$.

Sufficiency: Conversely, assume that $G \subseteq int_\delta(A)$, whenever G ⊆ A and G is (Λ, δ)-closed in (X, τ). Let $A^c \subseteq F$, where F is (Λ, δ)-open. Then $F^c \subseteq A$. By criteria, $F^c \subseteq int_\delta(A) \Longrightarrow cl_\delta(A^c) \subseteq F$. Thus A^c is λ_g^δ-closed and hence A is λ_g^δ-open.

Proposition 4.4: If $int_\delta(A) \subseteq B \subseteq A$ and A is λ_g^δ-open in (X, τ), then B is λ_g^δ-open in (X, τ).

Proof: It follows from Lemma 4.2 and Proposition 3.19.

Theorem 4.5: If A is λ_g^δ-open in X if and only if G = X whenever G is (Λ, δ)-open and $int_\delta(A) \cup A^c \subseteq G$.

Proof: Necessity: Let A be λ_g^δ-open set and G be (Λ, δ)-open and $int_\delta(A) \cup A^c \subseteq G$. This implies $G^c \subseteq (int_\delta(A) \cup A^c)^c = (int_\delta(A))^c \cap A = (int_\delta(A))^c \setminus A^c = cl_\delta(A^c) \setminus A^c$. Since A^c isλ_g^δ-closed and G^c is (Λ, δ)-closed by Theorem 3.15, it follows that $G^c = \phi$ and hence G = X.

Sufficiency: Suppose that F is (Λ, δ)-closed and F ⊆ A. Then $int_\delta(A) \cup A^c \subseteq G \subseteq int_\delta(A) \cup F^c \subseteq G$. Since δ-open ⟹ (Λ, δ)-open, we get $int_\delta(A)$ is (Λ, δ)-open and F^c is (Λ, δ)-open. Hence $int_\delta(A) \cup F^c$ is (Λ, δ)-open. By hypothesis, $int_\delta(A) \cup F^c = X$ and hence $F \subseteq int_\delta(A)$. Therefore by Proposition 4.4, A is λ_g^δ-open in X.

Proposition 4.6: For each a ∈ X either {a} is (Λ, δ)-closed or {a} is λ_g^δ-open in (X, τ). That is, for any space (X, τ), X = (Λ, δ)C(X, τ) ∪$\lambda_g^\delta O(X, \tau)$.

Proof: Suppose that {a} is not (Λ, δ)-closed then $\{a\}^c$ is not (Λ, δ)-open and the only (Λ, δ)-open set containing $\{a\}^c$ is the space X itself. That is, $\{a\}^c \subseteq X$. Therefore, $cl_\delta(\{a\}^c) \subseteq X$ and so $\{a\}^c$ is λ_g^δ-closed and hence {a} is λ_g^δ-open.

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